Physics Internal Assessment

How does the amount of water affect the frequency of the sound emitted by a singing wine glass?

Introduction

A singing wine glass if often considered a musical instrument. It produces sounds by standing waves with an open end and a closed end. Standing waves are present in many musical instruments, such as a Guitars or Sitars. Much like any other musical instrument, A singing wine glass doesn't only emit 1 sound, it can emit different kinds of sounds or to be more specific, sounds of different pitches. A singing wine glass's frequency changes depending on the amount of liquid in it. I am passionate about music which is why this topic of research appealed to me. I will be investigating the relationship between the amount of water in a wine glass, and the frequency produced it.

Theory

When you rub your finger, after dipping it in some water, around the rim of a wine glass, there is a slight vibration created due to the friction between your finger and the rim. This is referred to as the 'slip and stick' motion. This makes the side of the wine glass vibrate, in other words, gain kinetic energy, which is transmitted to the water molecules which in return make the air molecules vibrate, which is what produces the sound. This represents a standing wave, with one closed end and one open end. The surface of the water being the closed end and the open wine glass being the open end. As more liquid is added, there are more water molecules present which helps slow down the vibrations, resulting in a lower frequency due to the vibrations of the wine glass having to move a larger amount of molecules, which is heavier and therefore requires more kinetic energy to move. The relevant equation which I will be using for this investigation is the following(1) :

Original Equation :

$$\left(\frac{Fo}{Fd}\right)^2 = 1 + \left(\frac{B \times Pl \times R}{5 \times Pg \times A}\right)\left(1 - \frac{D}{H}\right)^4$$

The variables of the equation are explained in **Table 1**. Essentially, what I will be testing is the relationship between the distance from the top of the water to the top of the glass, and the frequency emitted by the singing wine glass at the relative water level. Representing this relationship graphically with the help of this equation, we would have $(Fo/Fd)^2$ on the Y axis and *D* on the X axis. However, this would not give us a linear relationship. Therefore, in order to obtain a positive linearised final graph, we would have $(Fo/Fd)^2$ on the Y axis and $(1-(D/H))^4$ on the X axis.

Table 1 : Variables of the original equation explained

Variable / Constant	Meaning
Fo	Frequency of empty wine glass
Fd	Frequency of partially filled glass
B	A Constant
Pl	Density of Liquid
R	Radius of water
Pg	Density of Glass
A	Glass's thickness
D	Distance from the top of the water to the top of the glass
H	Effective height of glass

Therefore, my hypothesis is that graphing $(Fo/Fd)^2$ against $(1-(D/H))^4$ will give me a positive linear relationship. The fundamental frequency is the frequency of the first harmonic and is typically the loudest frequency in musical instruments. The relationship between the harmonics and the frequency, F, is as follows :

1st Harmonic : F

2nd Harmonic : 2F

3rd Harmonic : 3F

So the difference between the frequency peaks (harmonics) during the frequency analysis represents the fundamental frequency(2). Hence, I will be taking the average of the differences between the peaks of the recorded sound waves in order to get an accurate reading of the fundamental frequency and be able to compare the different frequencies.

Equipment List

- A wine glass
- A measuring cylinder of 300 ml
- A mic (Should be built in to the laptop)
- A digital thermometer
- A 30 Centimeters Ruler

- Water (as much as needed)
- Audacity Software (3)

Method

1) Measure out 30 millilitres of water using the measuring cylinder.
2) Pour it into the wine glass, ensure that the wine glass doesn't already have any residue of water in it.
3) Check the temperature of the water using the thermometer to ensure that all the recordings are conducted by using water at the same temperature.
4) Then open audacity and press record.
5) Use 1 hand to stabilise the wine glass from the bottom, then use your other hand to circle the rim by using your index finger after dipping the finger in some water in order to make it is slightly wet for reduced friction. It might take some practice for one to be able to easily make the noise.
6) Upon circling your finger around the rim a few times and after you have been making the wine glass sing constantly for at least 5 seconds, you may stop the recording.
7) Measure the distance from the top of the water to the top of the glass using the ruler, make sure to measure it at eye level in order to avoid parallax errors.
8) Repeat steps 1 to 7, 5 times, in order to have sufficient repeats, use the best 3 recordings out of the 5, since recordings can often be faulty. Export the recordings on to a folder neatly for the analysis.
9) Repeat steps 1 to 8 with 60ml, 90ml, 120ml, 150ml, 180ml, 210ml, 250ml of water using the measuring cylinder and once with an empty wine glass.

Figure 1 shows a simple model of the set up of the experiment.

Figure 1 : Experiment set up

Variables

The independent variable is the distance from the top of the water to the top of the glass at different water levels. The dependant variable is the frequency of the sound waves emitted by the singing wine glass at different water levels. The controlled variables are listed in the 'Evaluation : Strengths' section on page 10.

Safety Concerns

- Wear a lab coat in order to protect any water spilling on your clothing.
- Handle the wine glass carefully, it may break since it's rather delicate.
- Incase the wine glass does break, call a supervisor or a superior and wear gloves and use necessary cleaning equipments to clean it up.
- You must be careful with your digital equipments also such as your laptop, if water spills on it, it may ruin your computer.

Data Processing

The Data Processing section will be divided in 2 sub sections, going chronologically. The first one being Frequency Analysis, where I explain how to calculate the fundamental frequency from the recordings. The second one being Further Data Processing, where I derive the final data points and plot a graph. Uncertainty calculations are included in both of these sub sections.

Data Processing 1 : Frequency Analysis

In order to deduce the fundamental frequency, I imported ever single one of my recordings into Audacity and I took the best 3 recordings, the recordings which have the least background noises. Then upon importing the recording, I would have to select a part of the recording where the wine glass was constantly 'singing' then click on 'Analyse' on the top of the screen then click on 'plot spectrum'. All the default settings for the spectrum shall remain, apart from the 'Axis' which you change to 'Linear frequency' if it isn't already chosen. So the spectrum of the frequency analysis should look more or less like **Figure 2**.

We can see that there are vaguely equidistant peaks on the spectrum on the left hand side, which is what I need to measure as they represent the harmonics. Upon bringing the cursor on the peak, it will give the frequency at the peak. Therefore, I get the frequency of 5 consecutive peaks in order to get a few differences between the peaks to average it and get the fundamental frequency, as explained previously in the theory section. I will show the results for the data extrapolation of the empty singing wine glass, 0 ml of water, in order to solidify my explanation.

Figure 2 : Frequency Analysis Spectrum

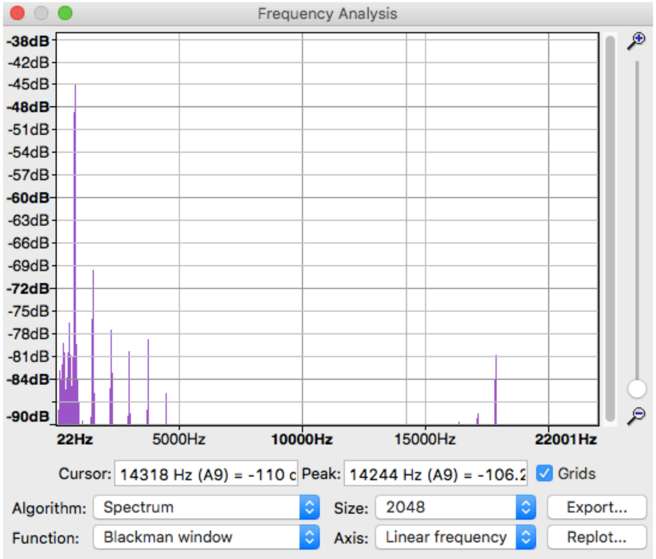


Table 2 shows the frequencies of the consecutive peaks of the spectrum in the frequency analysis of when the wine glass is empty.

Table 2 : Frequencies of consecutive peaks of the empty wine glass

Recording 1 (Hz)	Recording 2 (Hz)	Recording 3 (Hz)
742	742	744
1484	1487	1479
2232	2230	2225
2890	3718	2949
3701	4460	3719

The next step is taking the differences of the peaks, this is shown in **Table 3.** Example calculation for the first 2 peaks in recording 1 :

=> 1484 - 742 = 742 Hz

Table 3 : The differences between the consecutive peaks of the empty wine glass

Recording 1 (Hz)	Recording 2 (Hz)	Recording 3 (Hz)
742	745	735
745	743	746
658	1488	724
811	742	740

Moving on, we must now take the averages of the differences, which is presented in **Table 4.** In order to take the average of the differences between the peaks, you must take the sum of all the differences and divide it by the total number of differences.

For example for recording 1 : (742 + 745 + 658 + 811) / 4

=> 2’956/4

= 739 Hz

Table 4 : Averages of the differences between the consecutive peaks of the empty wine glass

Recording 1 (Hz)	Recording 2 (Hz)	Recording 3 (Hz)
739.0	743.3	736.5

However, we can see that there is an obvious anomaly in **Table 3**. It is the value of ‘1488 Hz’ on the 3rd row for Recording 2. It is extremely far off the other differences as all the other differences are around 740 Hz instead. So I did not count that while taking the average for recording 2. This anomaly is most likely caused by a missing harmonic.

Now in order to arrive at our final average, which will be the fundamental frequency of the sound waves emitted by the singing glass with 0 ml of water ; we must take the average of the averages. Therefore :

=> (739.0 + 743.3 + 736.5) / 3

=> 2’218.8 / 3

= 739.6 Hz is the fundamental frequency of the empty wine glass

Frequency Analysis : Uncertainties

Now in order to derive the uncertainty for the fundamental frequency, I will be taking the largest difference and subtracting the smallest difference from it, from **Table 3**, then dividing that answer by the total number of differences that I collected. The total number of differences which I collected is 11 (not counting the anomaly, since it was not included in the averages in the first place) The largest difference is 811 Hz and the smallest difference is 658 Hz so :

=> (811 - 658) / 11

=> 13.9 Hz

The fundamental frequency of the empty wine glass with the uncertainty, to 1 Significant Figure (S.F) is (740 +/- 10) Hz. This method was used to derive the frequencies at all the other water levels also, the results are in **Table 5.**

Table 5 : Fundamental Frequency relative to changes in the distance from the top of the water to the top of the glass

The distance from the top of the water to the top of the glass (cm) (+/- 0.05)	Fundamental Frequency (hertz) (+/- 10)	Volume of Water (ml) (+/- 2)
8.5	741	30
7.6	737	60
6.5	732	90
6.0	722	120
5.5	707	150
5.0	695	180
4.3	683	210
3.7	662	250

Data Processing 2 : Further Data Processing

Now that I've shown how to obtain the final frequencies, let's take a look at how to do the further calculations. I will be using the water level at 30 ml as an example. Processing the independent variable first :

$(1 - (D / 23))^4$

The value for D at 30 ml is 8.5 cm, substituting it, we get the following :

=> (1 - (8.5 / 23))

=> (0.63)^4

=> 0.158

We only have the evaluate the frequency now, we've already seen how to derive the frequency, now let's take a look took at processing the dependant variable :

$(Fo/Fd)^2$

The Fo is what we had previously derived, the frequency emitted by the singing wine glass when it's empty, at 0 ml of water and it stays constant at 740 Hz. The variable which changes is Fd since it's the frequency at different water levels, I'm going to take 30 ml of water as an example again for this and the frequency at 30 ml was 741 Hz :

=> (740/741)^2 = 0.997

Further Data Processing : Uncertainties

Let's do the *independent variables's* uncertainty first, with 30 ml of water like previously done. The uncertainty of the ruler is 0.05 centimetres. We need to convert this to percentage first :

=> (0.05/23) x 100 = 0.2%

=> (0.05/8.5) x 100 = 0.6%

Now, we need to take the sum of the percentage uncertainties, which is 0.8% and multiply by it 4 because of the exponent. Then apply it back to the original value in order to arrive at an absolute uncertainty :

=> 0.8 x 4 = 3.2%

=> 0.032 x (0.158) = **0.2 +/- 0.005**

We needed to round up the final Value to 1 significant figure (S.F) because we give the absolute certainty in 1 S.F also.

The uncertainty calculations for the *dependant variable* with 30 ml of water :

=> 10/740 = 1.35%

=> 10/741 = 1.35%

The sum of the percentage uncertainties is 2.7%, we need to multiply this by 2 because of the exponent then convert it into an absolute uncertainty :

=> 2.7 x 2 = 5.4%

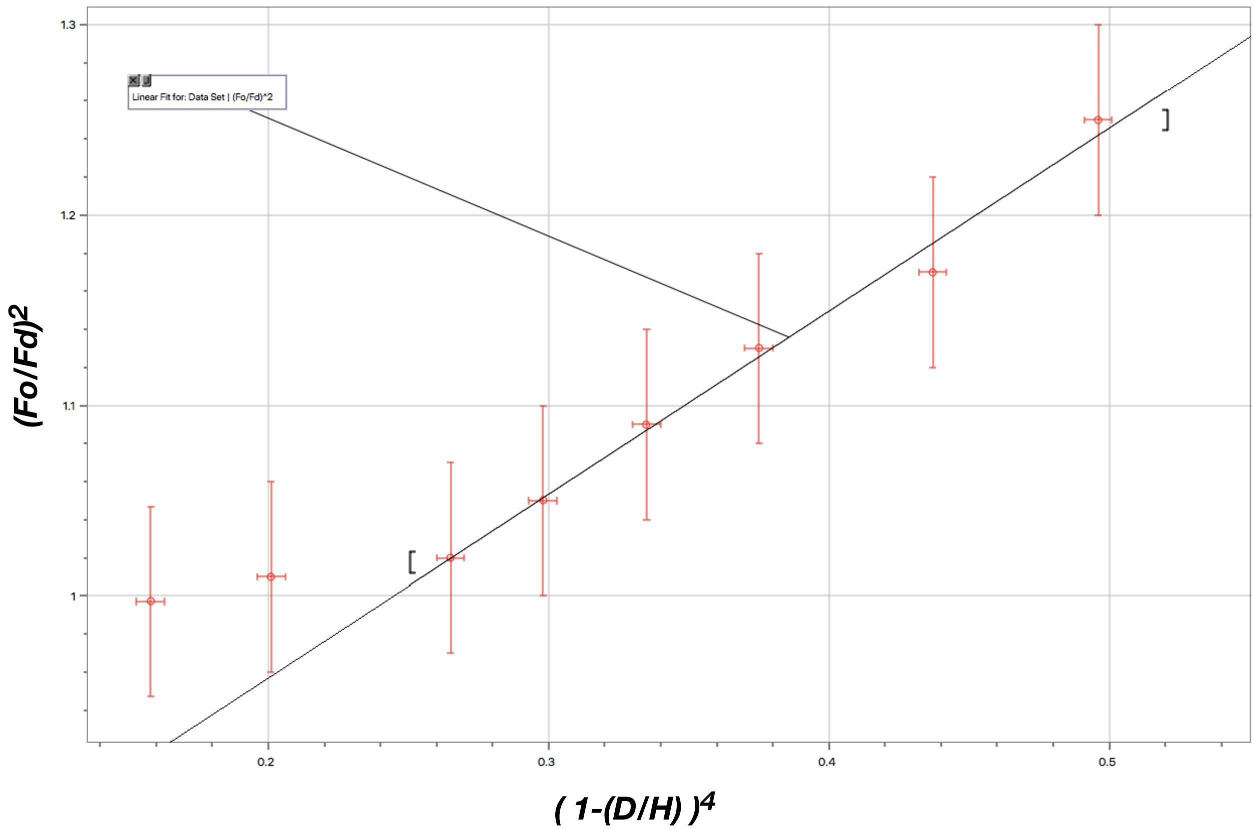
=> 0.054 x 0.997 = **1 +/- 0.05**

The final processed data is on **Table 6.** The graph obtained from the final data, graphed on Logger Pro(4) is on **Figure 3.**

Table 6 : The Final Processed Data

$(Fo/Fd)^2$ (+/- 0.005)	$((1-D)/H)^4$ (+/- 0.05)
0.158	1.00
0.201	1.01
0.265	1.02
0.298	1.05
0.335	1.09
0.375	1.13
0.437	1.17
0.496	1.25

Figure 3 : Final Graph for *(Fo/Fd)²* against *(1-(D/H))⁴*

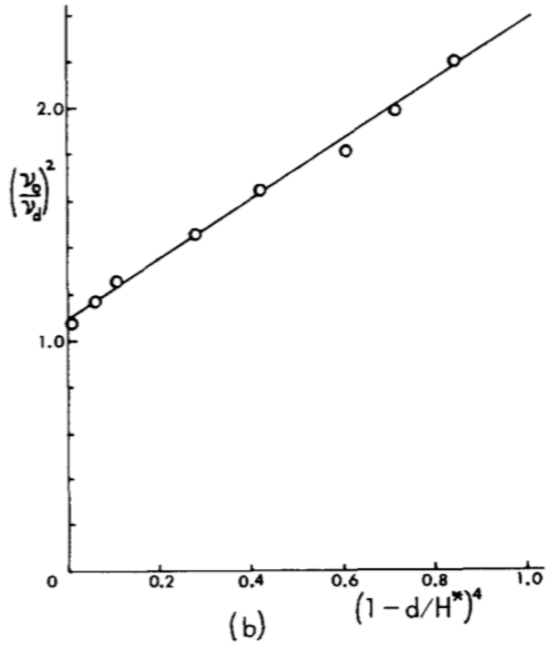


Conclusion

Taking the final results, **Table 6** and **Figure 3**, into consideration, my hypothesis was true, the investigation was successful. The relationship between $(Fo/Fd)^2$ and $(1-(D/H))^4$ is visibly positive and linear. However, the first 2 data points don't correlate with this relationship, which is why they are the only 2 data points that I didn't include in the line of best since I decided to treat them as anomalies. The original equation which I used for this investigation is meant for perfectly cylindrical glasses, which a wine glass is not. A wine glass is curvy instead of being perfectly cylindrical. Therefore, this effects the radius of the water, R, which is a variable in the original equation which was meant to stay constant. This could not be kept constant as the radius of the water changes according to the curvature of the wine glass. Hence, this could mean that perhaps if the amount of water is between 30 ml and 60 ml in a typical wine glass, then changing the amount of water in that range has very little to almost no effect on the frequency of the sound waves emitted from the singing wine glass.

A study was conducted on wine glass acoustics from Massachusetts Institute of Technology, Cambridge(1). They created the very same graph as I did. Their graph is on **Figure 4** for the sake of comparing my results with other professional investigations on the same topic. Both the graphs look fairly similar, except their graph doesn't seem to have any anomalies and their data points have a much larger range than mine. However, both are positive and linear.

Figure 4 : Graph for comparison



Evaluation

The Evaluation Section is divided into 3 subsections, going chronologically. The first one is strengths, where I discuss why this investigation was well conducted. The second sub section is limitations, where I discuss what lacked in this investigation and the potential sources of errors. The final subsection is the extension of this investigation, where I discuss other potential factors which could be investigated related to this topic.

Evaluation 1 : Strengths

There were many controlled variables which needed to remain constant, otherwise it could interfere with the results which I was trying to obtain. Temperature was one of the controlled variables. Since temperature is a measurement of the average kinetic energy of a body, a high temperature would mean the water in the glass would have more kinetic energy which mean the molecules in water would be vibrating a lot faster. Since the sound produced be the singing wine glass is dependant upon the vibrations of the glass and the water molecules, it would interfere with the results. Which is why I measured the temperature of the water before every recording and took the water from the same source (tap) every time.

The rest of the controlled variables were the specifications of the wine glass. This means that the density of the glass, the total height and the curvature must all remain constant since the design of the wine glass itself can also effect the pitch of the sound waves emitted by it. The total height of the glass, H, is very important especially since it is used in the equation which we used, *(1-(D/H))⁴.* The glass density is also in the full original equation, page 1. The radius of the water and the glass density and thickness is also used in the equation. These factors were controlled by using the same wine glass every time, apart from the radius of the water, further explained in the 'Limitations' sub section.

Moreover, the distance of the wine glass to the mic is also important. I marked a spot 15 cm away from the mic of my laptop to make sure that the distance of the wine glass to the mic is always constant too. Since sound waves lose amplitude the further they travel, so I kept the distance constant for fairness.

Evaluation 2 : Limitations

There are a number of ways in which one could improve the accuracy of this experiment. The significant aspect which I believe could be improved, is the speed at which one rotates their finger around the wine glass' rim in order to make it sing. This could prove to be rather difficult to keep constant so I tried to rotate my finger around the rim at the same speed as much as I could.

Another limitation could be the poor quality of the mic being used, I did originally get a proper studio microphone to take the recordings but unfortunately due to technical issues I could not use it, so I had to resort to using the microphone on the laptop.

Finally, the main limitation of this experiment was the radius of the water. Ideally I would have to keep the radius of the water constant as it is one of the variables in the original equation. However, the radius of the water can't be kept constant due to the curvature of the wine glass. This could also be a possible explanation for the 2 anomalies in **Figure 3**. This is a methodological error.

Evaluation 3 : Extension of the investigation

There are several possible different approaches to this experiment. For example like I had previously discussed, the temperature could potentially affect the frequency due to the excess kinetic energy added, or lack there of. So one could keep the distance from the top of the water to the top of the glass constant and make the temperature of water the independent variable. Similarly one could investigate with other independent variables while keeping the dependant variable the same. These could be the wine glass's thickness, the density of the liquid in the wine glass or the density of the wine glass used for example.

An alternate methodology could be used for a similar investigation. Instead of rotating one's finger around the wine glass at different water levels. One could tap the wine glass with a metal spoon instead for example. Then investigate how the frequency of the wine glass changes at different water levels when tapped by a metal spoon.

Bibliography :

1) A study of wine glass acoustics, Physics Department, Massachusetts Institute of Technology, Cambridge : https://www.nikhef.nl/~h73/tgo/praktgeluid/French1983.pdf

2) Second Edition, Physics Textbook for the IB Diploma by John Allum and Christopher Talbot by Hodder Education

3) Audacity Software : https://www.audacityteam.org

4) Logger pro : https://www.vernier.com/products/software/lp/